Dynamic bedrock channel geometry

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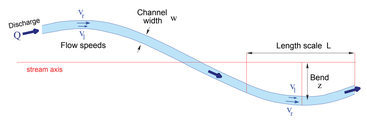
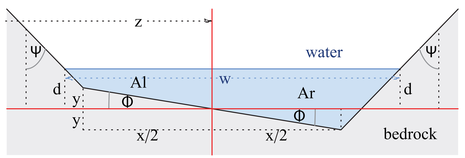
The evolution of many mountain landscapes is controlled by the incision of bedrock river channels. While the rate of incision is set by channel shape through its mediation of flow, the channel shape is itself set by the history of bedrock erosion. This feedback between channel geometry and incision determines the speed of landscape response to tectonic or climatic forcing. Here, a model for the dynamics of bedrock channel shape is derived from geometric arguments, a normal flow approximation for channel flow, and a threshold bed shear stress assumption for bedrock abrasion. The model dynamics describe the competing effects of channel widening, tilting, bending, and variable flow depth. Transient solutions suggest that channels may take ~1-10 ky to adapt to changes in discharge, implying that channel disequilibrium is commonplace. If so, landscape evolution models will need to include bedrock channel dynamics if they are to probe the effects of climate change.



Impose: *p(Q)*
discharge distribution

$$Q = A_l v_l + A_r v_r \qquad A_c = A_l + A_r$$

Track: *x, y, z, p(d), S*
width, tilt, bend, depths, slope



Don't assume flow width scales with mean discharge

$$v_l = (1-p)\,v \qquad v_r = (1+p)\,v$$

Solve for: shear stresses τ

Normal flow approx - different shear stresses across bed & bank - frictional shear scales with velocity squared

$$\tau_f = \tau_l + \tau_{cl} + \tau_{cr} + \tau_r \qquad \tau_d = \rho\, g \frac{A_c}{P_w} S$$

$$\tau_j \propto v_j^2 \qquad \tau_d = \tau_f$$

$$\tau_f = \frac{\rho\, C_f\, v^2}{2}\left[1 + p^2 + \frac{32\, pyd}{8(d^2+y^2)+(x^2+4y^2)(1+\cos(2\psi))}\right]$$

Obtain: flow speed ***v***
and: *dynamic* hydraulic geometry ***R***

$$v \sim \sqrt{RS} \qquad S \approx \frac{\Delta h}{\sqrt{z^2+L^2}}$$

$$R = f_R[x, y, d, p \,|\, \psi] \qquad \phi = \arctan(2y/x)$$

Derive Chezy-like equation

Let the channel change *evolve*

$$E = \eta\,(\tau - \tau_c)^\alpha \quad \text{for} \quad \tau \geq \tau_c$$

Non-linear, threshold bed shear stress bedrock erosion law
Channel erosion is also channel deformation

Closure assumptions - stabilizing process Ed - inner vs outer flow speed perturbation

$$E_d = \eta\,\kappa_d \tan\phi$$
$$\xi = \cosh(\kappa_z p) - 1$$
$$p = \kappa_p z / L$$

Bedrock erosion drives channel shape change

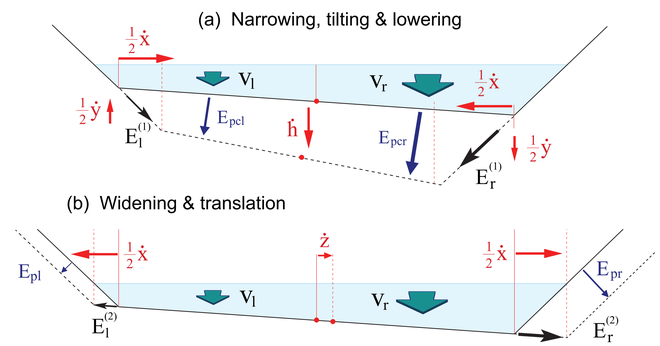
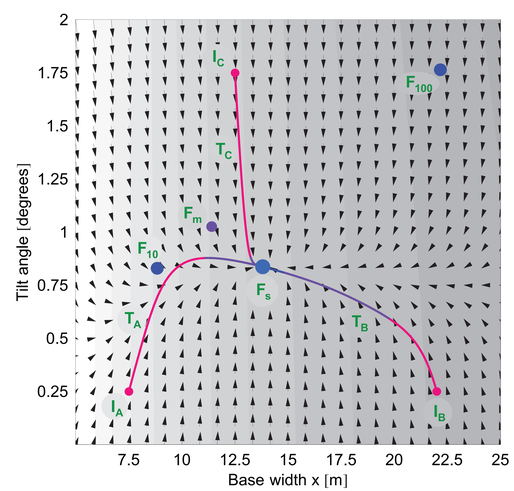


Illustration of the (a) narrowing and (b) widening mechanisms, and their relationships to the flow speeds *vl* and *vr*, the bank and base erosion rates *Epl, Epr, Epcl* and *Epcr*, and the erosion vectors *El* and *Er*.

Lowering rate
= mean rate of base incision

$$\dot{h} = \tfrac{1}{2}(\mathbf{E_r} + \mathbf{E_l}) \cdot \mathbf{k} = f_{\dot{h}}(x, y, z, d \,|\, \Omega_1)$$

Example model solutions: Channel shape equilibration

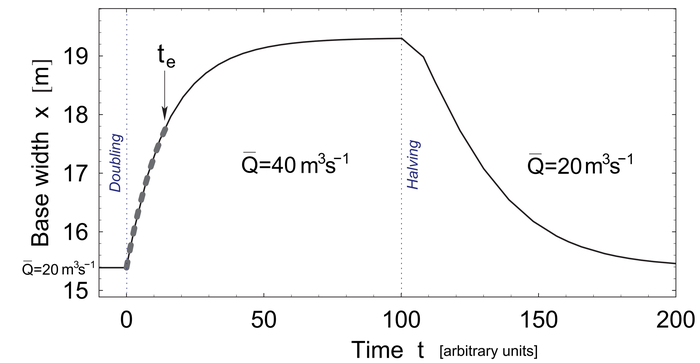


Solve the 3D dynamical system:
time evolution of width x, tilt y & "sinuosity" z

$$\begin{aligned}
\dot{x} &= (\mathbf{E_r} + \mathbf{E_l}) \cdot \mathbf{j} &&= f_{\dot{x}}(x, y, z, d \,|\, \Omega_1) \\
\dot{y} &= \tfrac{1}{2}(\mathbf{E_r} - \mathbf{E_l}) \cdot \mathbf{k} &&= f_{\dot{y}}(x, y, z, d \,|\, \Omega_1) \\
\dot{z} &= (\mathbf{E_r} - \mathbf{E_l}) \cdot \mathbf{j} &&= f_{\dot{z}}(x, y, z, d \,|\, \Omega_1) \\
S &= f_{s1}(x, y, z, d \,|\, \Omega_2) &&= f_{s2}(z \,|\, L, \Delta h)
\end{aligned}$$

Differential-algebraic equation

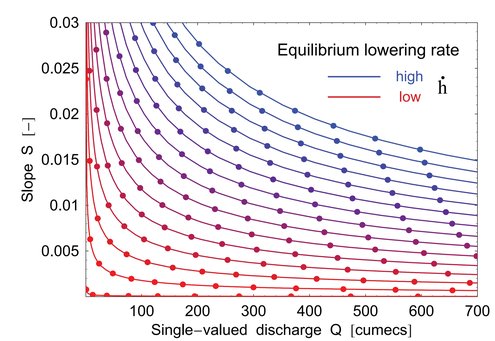
Climate change: channel shape adapts slowly



Equilibration time: Rule of thumb

E-folding time for channel base width ***x*** is approx the **time required to incise** vertically a distance ***x/10***.

So, a channel *x*=10m wide eroding at 0.1mm/y requires **~10ky** to **begin to approach equilibrium**.



If and when channel geometry equilibrates, simple scaling

$$\dot{h} = \mu\left(S\sqrt{Q}\right)^\gamma$$

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$$\mathbf{E_l} = \sec(\psi + \phi)\begin{pmatrix} E_{pl}\cos\phi - E_{pcl}(\sin\psi - \xi\sin\phi) \\ E_{pl}\sin\phi + E_{pcl}\cos\psi \end{pmatrix}$$

$$\mathbf{E_r} = \sec(\psi - \phi)\begin{pmatrix} E_{pr}\cos\phi - E_{pcr}(\sin\psi + \xi\sin\phi) \\ E_{pr}\sin\phi + E_{pcr}\cos\psi \end{pmatrix}$$

Just two vectors describe channel shape evolution

$$E_{pcl} = \chi\,\eta\left\{\left[\frac{2^{-1}(p-1)^2 v^2 (x^2+4y^2)}{x^2+4y^2+4(d^2+y^2)\sec^2\psi}\right]^\alpha + \frac{2\kappa_d y}{x}\right\} \qquad E_{pl} = 2^\alpha \eta\left(\frac{(p-1)^2 v^2 (d-y)^2}{(x^2+4y^2)\cos^2\psi + 4(d^2+y^2)}\right)^\alpha$$

$$E_{pcr} = \chi\,\eta\left\{\left[\frac{2^{-1}(p+1)^2 v^2 (x^2+4y^2)}{x^2+4y^2+4(d^2+y^2)\sec^2\psi}\right]^\alpha - \frac{2\kappa_d y}{x}\right\} \qquad E_{pr} = 2^\alpha \eta\left(\frac{(p+1)^2 v^2 (d-y)^2}{(x^2+4y^2)\cos^2\psi + 4(d^2+y^2)}\right)^\alpha$$